Internet Appendices for

"Debt Correlations in the Wake of the Financial Crisis: What are Appropriate Default Correlations for Structured Products?"

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Internet Appendix IA 1

Table IA.1
Additional Deal Summary Statistics
This table reports additional deal level summary statistics for CDOs in our sample. *p10* and *p90* denote the 10th and 90th percentiles, respectively.

	N	Mean	Median	Std. Dev.	p10	p90
No. of Obligors	1109	130.76	122.00	62.69	62.00	210.00
Deal Size ($M)	1096	398.16	375.00	334.30	0.48	707.00
WAM	1109	6.18	5.83	1.33	4.93	7.71
S&P SDR	867	0.361	0.359	0.120	0.200	0.503
Moody's SDR	867	0.334	0.332	0.110	0.228	0.439

Table IA.2
Correlation of Rating Changes

This table reports the correlation matrix of directional rating changes. Reported are the time-series correlations for the percentage of directional rating changes at the monthly level from January 1986 to December 2012.

	Rating Downgrade	Rating Unchanged	Rating Upgrade
Rating Downgrade	1		
Rating Unchanged	-0.885	1	
Rating Upgrade	-0.160	-0.319	1

Table IA.3
Post-Crisis Portfolio SDRs with Estimated Missing Values

This table reports the Scenario Default Rate (Panel B) for a sample of 279 corporate backed CDOs issued from 2011-2014. Missing SDRs not reported by S&P are estimated using fitted values after regressing SDRs reports from S&P on a Gaussian copula simulation. Reported are summary statistics for AAA SDRs estimated by S&P, under our methodology when modeling rating changes and frailty (*Ratings Only*) and under our methodology when modeling rating changes, macroeconomic covariates, and model frailty (*Ratings & Macro Covariates)*. The sample consists of collateralized loan obligations (CLOs) and collateralized bond obligations (CBOs) issued from June 2011 to June 2014.

Methodology	**N**	**Mean**	**Median**	**Std. Dev.**
CRA Assumptions:				
S&P's SDRs	282	0.617	0.604	0.038
Ratings Only:				
2-State HMM	282	0.820	0.823	0.028
3-State HMM	282	0.818	0.820	0.028
Continuous-State HMM	282	0.796	0.797	0.027
Ratings & Macro Covariates:				
2-State HMM	282	0.688	0.688	0.030
3-State HMM	282	0.691	0.691	0.030
Continuous-State HMM	282	0.696	0.696	0.030

Table IA.4
Pre-2007 ABS Parameter Estimates

This table reports the results of an exponential hazard model of defaults (Panel A) and estimates for the two-state HMM model of rating transitions (Panels B & C) for the Bloomberg universe of ABS from January 1990 to December 2006. '*Rating Implied Intensity*' is the implied default intensity from S&P's one-year rating default probabilities. '*AAA Spread*' is difference in AAA corporate debt yields and the 10-year Treasury Rate and '*3-Month Yield*' is the Treasury yield, both reported by FRED. '*Unemployment*' is the seasonally adjusted U.S. civilian unemployment rate. '*12-Month Market Return*' is the lagged annual CRSP value-weighted return. '*Frailty Volatility*' is the scaling factor, η, from equation (XX). '*Frailty Mean-Reversion*' is the speed of mean-reversion, κ, from equation (XX). Data is at the monthly level. Standard errors are reported in parentheses. Log-Likelihood is the average Log-Likelihood across all frailty paths drawn from the Gibbs sampler.

Panel A. Hazard Model Parameter Estimates

	(1)	(2)	(3)	(4)
Rating Implied Intensity	0.458	0.454	0.458	0.454
	(.0127)	(.0130)	(.0126)	(.0130)
AAA Spread			-0.960	-0.636
			(.1904)	(.2837)
3-Month Yield			0.117	0.005
			(.0177)	(.0167)
Unemployment			-0.004	0.017
			(.0011)	(.0650)
12-Month Market Return			-2.044	-2.185
			(.2773)	(.6561)
Frailty Volatility, η		0.754		0.703
		(.0176)		(.0208)
Frailty Mean-Reversion, κ		0.803		0.882
		(.0590)		(.0846)
Intercept	-2.912	-3.717	0.004	-0.590
	(.0967)	(.1076)	(.0030)	(.7559)
No Obs.	511101	511101	511101	511101
Log-Likelihood	-4172.58	-3538.09	-4087.71	-3519.26

Panel B. Two-State HMM: Rating Transitions

	AAA	AA	A	BBB	BB	B	CCC	CC
$E(Rating_{t+1} \mid Bad)$ - $E(Rating_{t+1} \mid Good)$	0.002	0.013	0.042	0.066	0.062	0.033	0.016	0.006
	(.0004)	(.0025)	(.0059)	(.0083)	(.0067)	(.0084)	(.0082)	(.0572)

Panel C. Two-State HMM: State Transitions

Current State:	Good	Bad
Prob(Good) at t+1	0.952	0.262
	(.0124)	(.0859)
Prob(Bad) at t+1	0.048	0.738
	(.0124)	(.0859)

Table IA.5
Full Sample ABS Parameter Estimates

This table reports the results of an exponential hazard model of defaults (Panel A) and estimates for the two-state HMM model of rating transitions (Panels B & C) for the Bloomberg universe of ABS from January 1990 to April 2014. '*Rating Implied Intensity*' is the implied default intensity from S&P's one-year rating default probabilities. '*AAA Spread*' is difference in AAA corporate debt yields and the 10-year Treasury Rate and '*3-Month Yield*' is the Treasury yield, both reported by FRED. '*Unemployment*' is the seasonally adjusted U.S. civilian unemployment rate. '*12-Month Market Return*' is the lagged annual CRSP value-weighted return. '*Frailty Volatility*' is the scaling factor, η, from equation (XX). '*Frailty Mean-Reversion*' is the speed of mean-reversion, κ, from equation (XX). Data is at the monthly level. Standard errors are reported in parentheses. Log-Likelihood is the average Log-Likelihood across all frailty paths drawn from the Gibbs sampler.

Panel A. Hazard Model Parameter Estimates

	(1)	(2)	(3)	(4)
Rating Implied Intensity	0.422	0.422	0.455	0.422
	(.0047)	(.0048)	(.0058)	(.0057)
AAA Spread			-0.215	-0.221
			(.0683)	(.0709)
3-Month Yield			-0.089	-0.186
			(.0409)	(.0278)
Unemployment			-0.167	-0.162
			(.0177)	(.0157)
12-Month Market Return			-1.143	-1.192
			(.0836)	(.0971)
Frailty Volatility, η		0.589		0.434
		(.0082)		(.0064)
Frailty Mean-Reversion, κ		0.465		0.446
		(.0316)		(.0334)
Intercept	-3.000	-3.566	0.004	-0.487
	(.0176)	(.0217)	(.0006)	(.3379)
No Obs.	977562	977562	977562	977562
Log-Likelihood	-27315.86	-25121.93	-26886.12	-25141.16

Panel B. Two-State HMM: Rating Transitions

	AAA	AA	A	BBB	BB	B	CCC	CC
$E(Rating_{t+1} \mid Bad)$ - $E(Rating_{t+1} \mid Good)$	0.052	0.082	0.086	0.089	0.086	0.072	0.028	0.001
	(.0021)	(.0036)	(.0034)	(.0031)	(.0039)	(.0032)	(.0015)	(.0005)

Panel C. Two-State HMM: State Transitions

Current State:	Good	Bad
Prob(Good) at t+1	0.961	0.212
	(.0042)	(.0226)
Prob(Bad) at t+1	0.039	0.788
	(.0042)	(.0226)

Table IA.6
MBS Parameter Estimates

This table reports the results of an exponential hazard model of defaults (Panel A) and estimates for the two-state HMM model of rating transitions (Panels B & C) for the Bloomberg universe of MBS from January 1990 to April 2014. '*Rating Implied Intensity*' is the implied default intensity from S&P's one-year rating default probabilities. '*AAA Spread*' is difference in AAA corporate debt yields and the 10-year Treasury Rate and '*3-Month Yield*' is the Treasury yield, both reported by FRED. '*Unemployment*' is the seasonally adjusted U.S. civilian unemployment rate. '*12-Month Market Return*' is the lagged annual CRSP value-weighted return. '*Frailty Volatility*' is the scaling factor, η, from equation (XX). '*Frailty Mean-Reversion*' is the speed of mean-reversion, κ, from equation (XX). Data is at the monthly level. Standard errors are reported in parentheses. Log-Likelihood is the average Log-Likelihood across all frailty paths drawn from the Gibbs sampler.

Panel A. Hazard Model Parameter Estimates

	(1)	(2)	(3)	(4)
Rating Implied Intensity	0.410	0.415	0.454	0.416
	(.0056)	(.0058)	(.0073)	(.0070)
AAA Spread			-0.004	-0.058
			(.0019)	(.1815)
3-Month Yield			-0.016	-0.127
			(.0312)	(.0347)
Unemployment			-0.163	-0.141
			(.1583)	(.0184)
12-Month Market Return			-1.523	-1.569
			(.0726)	(.1270)
Frailty Volatility, η		0.589		0.552
		(.0111)		(.0121)
Frailty Mean-Reversion, κ		0.465		0.502
		(.0469)		(.0549)
Intercept	-2.949	-3.431	-0.004	-0.435
	(.0211)	(.0266)	(.0626)	(.2173)
No Obs.	587283	587283	587283	587283
Log-Likelihood	-19151.83	-17802.84	-18670.40	-17804.43

Panel B. Two-State HMM: Rating Transitions

	AAA	AA	A	BBB	BB	B	CCC	CC
E($Rating_{t+1}$ | Bad) - E($Rating_{t+1}$ | Good)	0.051	0.080	0.115	0.114	0.096	0.064	0.030	0.000
	(.0018)	(.0033)	(.0043)	(.0038)	(.0041)	(.0028)	(.0018)	(.0007)

Panel C. Two-State HMM: State Transitions

Current State:	Good	Bad
Prob(Good) at t+1	0.936	0.261
	(.0052)	(.0269)
Prob(Bad) at t+1	0.064	0.739
	(.0052)	(.0269)

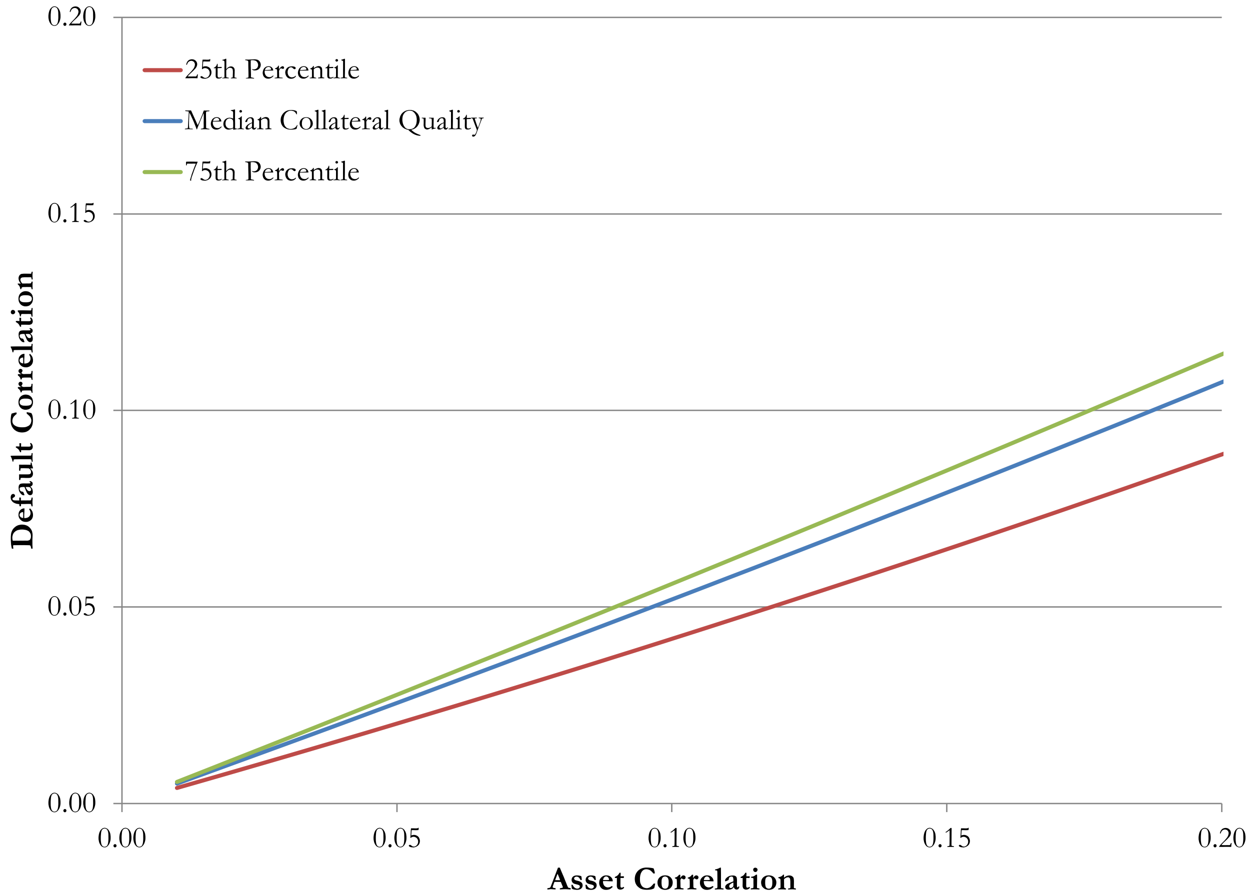

Figure IA.1
Asset Correlation versus Default Correlation
This figure illustrates the relationships between asset correlation and default correlation. The x-axis reports the asset correlation used as an input in the Gaussian Copula and the y-axis reports the average pair-wise default correlation from the simulated asset defaults. A pool of 122 assets, which corresponds to the median number of obligors in our sample of CDOs, was simulated 100,000 times for each asset correlation assumed. Reported are the mappings from asset correlation to default correlation when each assets probability of default is set to the 25th, 50th, and 75th percentiles of the average underlying collateral probability of default in our sample.

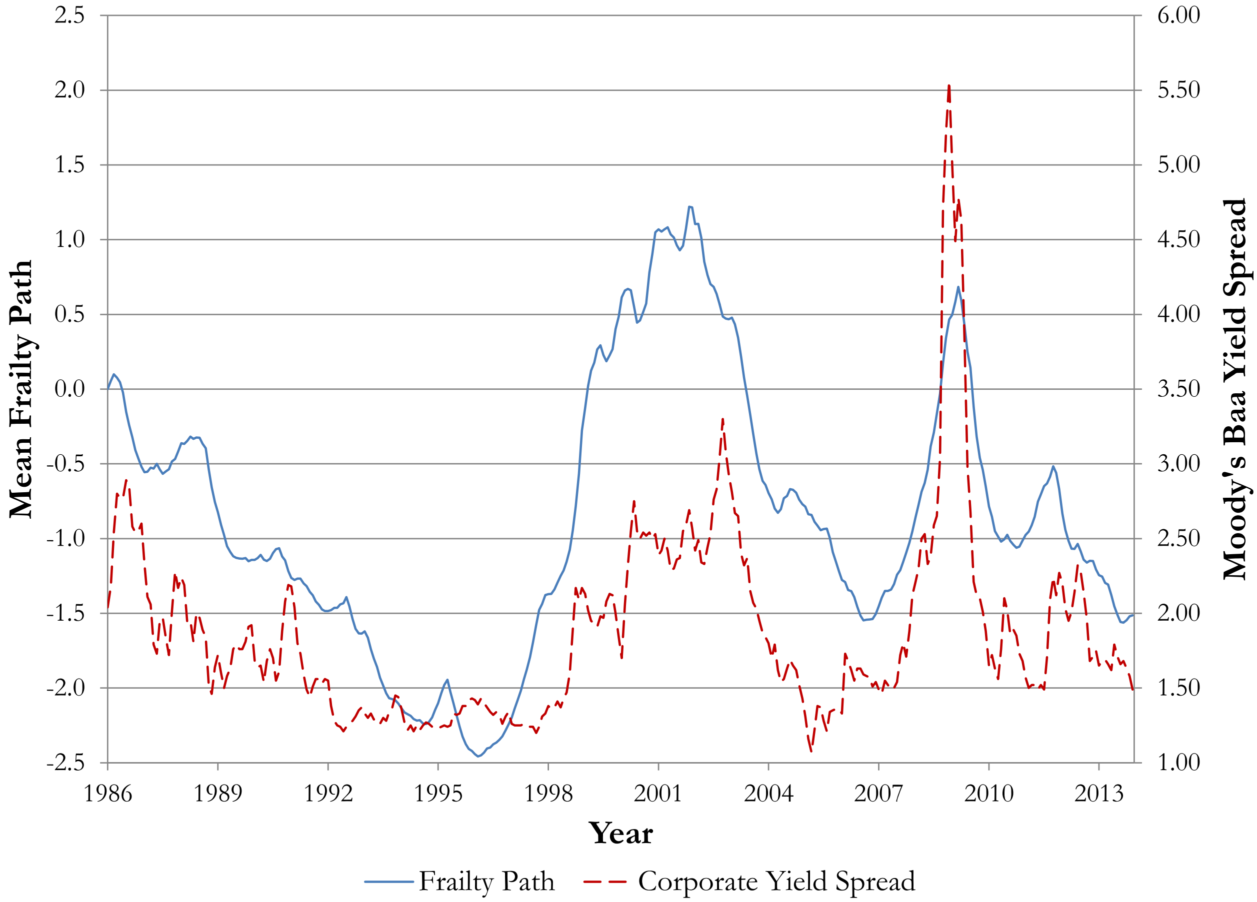

Figure IA.2
Corporate Bond Yield and Estimated Frailty Path
This figure illustrates the conditional mean of the frailty path from a hazard model fitted using the firm's credit rating lagged by 1 month (solid blue line) and the monthly corporate credit yield spread (dashed red line). 4,800 paths were drawn from a Gibbs sampler using the estimated coefficients from the fitted frailty and rating transition models reported in Table II. The path has been scaled by the appropriate scaling parameter, η. *Corporate Yield Spread* is the Moody's Seasoned Baa Corporate Bond Yield minus the 30-year Treasury Constant Maturity Rate, both provided by FRED.

Internet Appendix IA 2

This internet appendix details the data used (Section A), derives default correlation from the proprietary statistics reported by S&P and Moody's (Section B) briefly outlines the procedure to estimate the unobservable frailty path using the Expectations-Maximizing algorithm (Section C) and details the time-series estimation of the macro covariates (Section D).

A. Data

We obtain data for: a) rating agency issuance reports for CLOs and ABS CDOs, used to back out their default correlation assumptions, b) the rating and default histories of corporate bonds, and c) the rating and default histories of structured finance products. We briefly describe each type of data.

Our large sample of detailed CDO information is collected from the first available surveillance reports obtained from both S&P and Moody's, and supplemented by pre-sale and new issue reports. We only collect data from S&P and Moody's as Fitch is a much smaller player in the market over our sample period. We require the number of obligors from S&P's surveillance report when computing default correlations. This limitation results in a final sample of 1,109 deals rated by S&P and 1,064 deals rated by Moody's. Our sample consists of CDOs backed by bond and loan collateral and CDOs backed by ABS, typically non-agency housing (sub-prime, Alt-A, or prime) collateral either of high grade, or in most cases mezzanine quality. Further details of this data set can be found in Griffin, Nickerson, and Tang (2013). At the end of the paper, we also discuss results from recently collected S&P new issuance reports of a similar nature for CLOs originated from 2009 to 2014.

We obtain our corporate ratings and default histories from Compustat. Specifically, we obtain S&P's long term credit ratings updated at a monthly level for roughly 2,000 firms from

January 1, 1986 to December 31, 2013. Additionally, primary default information is obtained from the deletion date and reason fields provided by CRSP/Compustat.

We collect rating histories for all structured finance products from Bloomberg from 1986 through April 2014. The 207,661 covered tranches generally consist of various types of CLOs, MBS, ABS, and CDOs. We construct the default history on these assets by classifying a tranche in default if its credit rating is downgraded to 'D' or if the tranche realizes any losses and subsequently has its credit rating withdrawn.

B. Default Correlation

The variance of the percentage of defaults in a pool of assets, $Var(D_P)$ is:

$$Var(D_P) = \sum_{i=1}^{N} h_i \cdot Var(D_i) + \sum_{i=1}^{N}\sum_{j\neq i}^{N} h_i \cdot h_j \cdot Cov(D_i, D_j)$$

$$= \sum_{i=1}^{N} {h_i}^2 \cdot Var(D_i) + \sum_{i=1}^{N}\sum_{j\neq i}^{N} h_i \cdot h_j \cdot \sqrt{Var(D_i)Var(D_j)}\rho_{i,j}$$

where D_i is the default of asset i, and h_i is the percentage of asset i's size relative to the total asset pool. In the case of equal sized assets and equal probabilities of default, the assets become homogeneous with respect to their variances, $Var(D_k)$, and the equation simplifies to:

$$= \frac{1}{N} \cdot Var(D_k) + \sum_{i=1}^{N}\sum_{j\neq i}^{N} \frac{1}{N \cdot N} \cdot Var(D_k)\rho_{i,j}$$

Therefore, substituting the Average Correlation (ρ)

$$= \frac{1}{N} \cdot Var(D_k) + \frac{N-1}{N} \cdot Var(D_k)\rho$$

$$= \left[\frac{1}{N} + \frac{N-1}{N} \cdot \rho\right] Var(D_k)$$

In the case of Moody's correlation metric, the Diversity Score is such that the variance of a pool of DS independent and equal sized assets is:

$$\frac{1}{DS} \cdot Var(D_k)$$

Therefore the average correlation should be such that both portfolios have the same variance:

$$\frac{1}{DS} \cdot Var(D_k) = \left[\frac{1}{N} + \frac{N-1}{N} \cdot \rho\right] Var(D_k)$$

$$\frac{1}{DS} = \frac{1}{N} + \frac{N-1}{N} \cdot \rho$$

$$\frac{N-DS}{DS} = (N-1) \cdot \rho$$

$$\rho = \frac{N-DS}{DS \cdot (N-1)}$$

In contrast, S&P's correlation measure, CM, is defined as the ratio of the standard deviation of the portfolio defaults with pair-wise correlations to the standard deviation of the portfolio without any correlations:

$$CM = \frac{SD(Port\ with\ Corr.)}{SD(Port\ without\ Corr.)}$$

$$CM^2 = \frac{Var(Port\ with\ Corr.)}{Var(Port\ without\ Corr.)}$$

With equal sized assets, this equality reduces to:

$$CM^2 = \frac{\frac{1}{N} \cdot Var(D_k) + \frac{N-1}{N} \cdot Var(D_k)\rho}{\frac{1}{N} \cdot Var(D_k)}$$

$$= \frac{\frac{1}{N} + \frac{N-1}{N} \cdot \rho}{\frac{1}{N}}$$

$$CM^2 = 1 + (N-1) \cdot \rho$$

$$CM^2 - 1 = (N-1) \cdot \rho$$

$$\rho = \frac{CM^2 - 1}{N-1}$$

Note: Both of these derivations use the assumption that the underlying assets are equal sized, which may be slightly restrictive. However, it should be noted that since the purpose of a CDO is to diversify risk, the most efficient way to do this with N assets is to choose assets of equal sizes.

C. Estimating the Frailty Path Using the E-M Algorithm

This section briefly outlines the steps used to estimate the unobservable frailty path. We follow the procedure laid out in Duffie, Eckner, Horel and Saita (2009), which should be consulted for a more thorough discussion of the estimation procedure.

The E-M algorithm involves iterating between two steps until convergence of the parameters is achieved. To begin, we initialize the parameter set $\Theta = \{\alpha, \beta, \eta, \kappa\}$ to $\Theta^{(0)} = \{\hat{\alpha}, \hat{\beta}, .1, .05\}$ where $\hat{\alpha}$ and $\hat{\beta}$ are the MLE estimates from the model in equation (1) excluding the frailty component for our sample of observable covariates and firm defaults, Z. We then iterate between an expectation step (E) and a maximization step (M), starting with the first iteration ($i = 1$) in the following manner:

<u>*E Step*</u>

Draw N sample paths for the un-observed frailty path Y from the conditional probability of Y given the parameter set $\Theta^{(i-1)}$ using the following Gibbs sampler:

1. Initialize the frailty path, such that $\{Y_1^{(0)}, \dots, Y_T^{(0)}\} = 0$
2. Set k = 0
3. Draw a candidate path $y = \{y_1, \dots, y_T\}$, such that $y_t \sim N(Y_t^{(k)}, 4)$
4. Compute the acceptance probability for each period of the candidate path, $\alpha_1, \dots, \alpha_T$ in the following manner:

$$\alpha_t = min\left(\frac{\mathcal{L}\left(\Theta|\mathrm{Z}, \mathrm{Y}_{(-\mathrm{t})} = \mathrm{Y}_{(-\mathrm{t})}^{(\mathrm{k})}, \mathrm{Y}_\mathrm{t} = \mathrm{y}_\mathrm{t}\right)}{\mathcal{L}\left(\Theta|\mathrm{Z}, \mathrm{Y}_{(-\mathrm{t})} = \mathrm{Y}_{(-\mathrm{t})}^{(\mathrm{k})}, \mathrm{Y}_\mathrm{t} = \mathrm{Y}_\mathrm{t}^{(\mathrm{k})}\right)}, 1\right)$$

5. Generate the next draw of the frailty path using the acceptance probabilities calculated in step 5 as follows:

$$Y_t^{(k+1)} = \begin{cases} y_t & \text{if } u_t < \alpha_t \\ Y_t^{(k)} & \text{otherwise} \end{cases}$$

$$u_t \sim Uniform(0,1)$$

6. Replace k = k + 1. Return to step 3.

Note: We use a burn-in sample of 1,000 draws which we discard before drawing paths from the Gibbs sampler.

<u>*M Step*</u>

Compute the parameter set $\Theta^{(\mathrm{i})}$ which maximizes the expected log-likelihood of equation (3) using the sample frailty paths, $\{Y^{(1)}, \dots, Y^{(N)}\}$:

$$\Theta^{(\mathrm{i})} = \arg\max_{\Theta} E[\log \mathcal{L}(\Theta|\mathrm{Z}, \mathrm{Y})]$$

$$= \arg\max_{\Theta} \frac{1}{N}\sum_{i=1}^{N} \log \mathcal{L}\left(\Theta|\mathrm{Z}, \mathrm{Y}^{(\mathrm{i})}\right)$$

We then check for convergence of the parameter set. If not achieved, replace $i = i + 1$ and repeat both the E and M steps.

D. Time-Series Modeling of Macro Covariates

To model the impact of macroeconomic covariates on the co-movement in default risk across firms, we must first model the time-series dynamics of the macro variables. We opt for a first-order autoregressive structure to describe each process.

Specifically, we model the civilian unemployment rate U_t and trailing one-year market return S_t at time *t* as independent AR(1) processes:

$$\begin{pmatrix} U_{t+1} \\ S_{t+1} \end{pmatrix} = \begin{pmatrix} \alpha_{\mathrm{U}} \\ \alpha_{\mathrm{S}} \end{pmatrix} + \begin{pmatrix} \rho_{\mathrm{U}} & 0 \\ 0 & \rho_{\mathrm{S}} \end{pmatrix} \begin{pmatrix} U_t \\ S_t \end{pmatrix} + \begin{pmatrix} \sigma_{\mathrm{U}} & 0 \\ 0 & \sigma_{\mathrm{S}} \end{pmatrix} \varepsilon_{t+1}$$

where ε is a two-dimensional vector of independent standard random normal variables. Over the full sample, we obtain the following parameter estimate:

$$\begin{pmatrix} \hat{\alpha}_{\mathrm{U}} \\ \hat{\alpha}_{\mathrm{S}} \end{pmatrix} = \begin{pmatrix} .0303 \\ .0970 \end{pmatrix} \qquad \begin{pmatrix} \hat{\rho}_{\mathrm{U}} \\ \hat{\rho}_{\mathrm{S}} \end{pmatrix} = \begin{pmatrix} .9942 \\ .9130 \end{pmatrix} \qquad \begin{pmatrix} \hat{\sigma}_{\mathrm{U}} \\ \hat{\sigma}_{\mathrm{S}} \end{pmatrix} = \begin{pmatrix} .1572 \\ .0696 \end{pmatrix}$$

For the 3-month interest rate and AAA corporate spread over the 10-year rate, we jointly model the three interest rates using a first-order vector autoregression with restrictions on some lagged coefficients as follows:

$$r_{t+1} = \begin{pmatrix} r_{t+1}^{\mathrm{3m}} \\ r_{t+1}^{\mathrm{10y}} \\ r_{t+1}^{\mathrm{corp}} \end{pmatrix} = \begin{pmatrix} \alpha_{\mathrm{3m}} \\ \alpha_{\mathrm{10y}} \\ \alpha_{\mathrm{corp}} \end{pmatrix} + \begin{pmatrix} \varphi_{11} & \varphi_{12} & 0 \\ \varphi_{21} & \varphi_{22} & 0 \\ 0 & \varphi_{32} & \varphi_{33} \end{pmatrix} r_t + L\eta_{t+1}$$

$$LL^T = \Sigma$$

where η_{t_1} is a three-dimensional independent standard random normal variable and L is a 3 x 3 lower triangular matrix so that LL^T is the covariance matrix Σ of innovations across the system of time-series. The full sample yields the following parameter estimates:

$$\begin{pmatrix} \hat{\alpha}_{\mathrm{3m}} \\ \hat{\alpha}_{\mathrm{10y}} \\ \hat{\alpha}_{\mathrm{corp}} \end{pmatrix} = \begin{pmatrix} -.0728 \\ .0243 \\ .1795 \end{pmatrix} \quad \hat{\varphi} = \begin{pmatrix} .9785 & .0237 & 0 \\ -.0015 & .9928 & 0 \\ 0 & .0585 & .9240 \end{pmatrix} \qquad \hat{\Sigma} = \begin{pmatrix} .0359 & & \\ .0404 & .0594 & \\ .0081 & .0200 & .0376 \end{pmatrix}$$

Note: All interest rates and the civilian unemployment rate are expressed in percentage terms before performing the estimation procedure.